Student's Name:________________________ Teacher's Code : ________________________

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2022

Year 12
Mathematics (Extension One)
Task 4
Trial HSC Examination
Date : 26th August 2022

General Instructions:

- **Reading time: 10 minutes**
- **Time Allowed: 2 hours**
- Write using black pen
- Calculators approved by NESA may be used
- Attempt all questions in the booklets provided
- Write **your name** and **your teacher's code** in the positions indicated
- Marks may not be awarded for missing or carelessly arranged working.

Teachers :

- Mr R Maxwell **REM**
- Mr D Reidy **DPR**
- Mr N Mushan **NHM**
- Mr J Newey **JPN**

Topics Examined:

Section A	**10 Marks**
Multiple Choice	
Section B	
Short Answer	
Question 11	**15 Marks**
Question 12	**15 Marks**
Question 13	**15 Marks**
Question 14	**15 Marks**
Total	**70 Marks**

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SECTION I: Multiple Choice Questions:

1. Given $f(x) = \sqrt{x} - 3$, what are the domain and range of $\boldsymbol{f^{-1}(x)}$?

   (A) $x \geq -3, y \geq 0$

   (B) $x \geq -3, y \geq -3$

   (C) $x \geq 0, y \geq 0$

   (D) $x \geq 0, y \geq -3$

2. What is the value of $\sin 2x$, given that $\sin x = 0.8$ and x is obtuse?

   (A) $-\dfrac{12}{25}$

   (B) $-\dfrac{24}{25}$

   (C) $\dfrac{12}{25}$

   (D) $\dfrac{24}{25}$

3. The graph of the function $y = \cos^{-1}(2x)$ is dilated horizontally by a scale factor of 4 and then translated vertically by 3 units.
   What is the new equation?

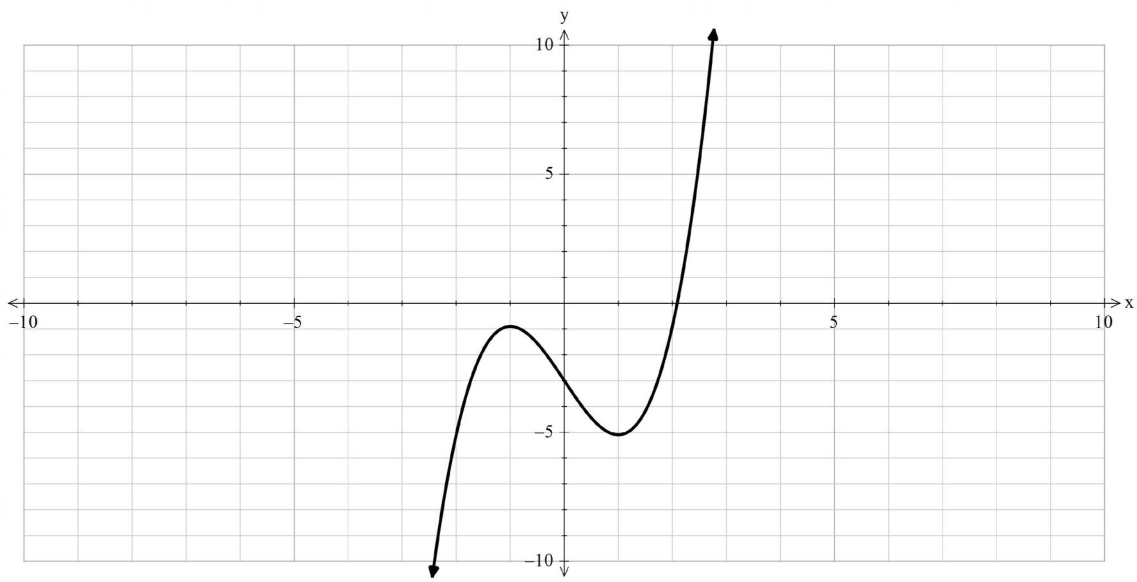
   (A) $y = \cos^{-1}\left(\dfrac{2x}{4} + 3\right)$

   (B) $y = \cos^{-1}(8x) + 3$

   (C) $y = \cos^{-1}\left(\dfrac{2x}{4}\right) + 3$

   (D) $y = \cos^{-1}(8x + 3)$

4. Consider the graph of $y = f(x)$ shown below:



Which one of the following would have 2 more roots than $f(x)$?

(A) $y = -2 \times f(x)$

(B) $y = f(x) + 3$

(C) $y = f^{-1}(x)$

(D) $y = f(x + 3)$

5. If $f(x) = \dfrac{3 + e^{2x}}{5}$ where $f(x) > \dfrac{5}{3}$ which of the following is $\boldsymbol{f^{-1}}(x)$?

(A) $\ln(5x - 3)$

(B) $\dfrac{1}{2}\ln(5x - 3)$

(C) $\ln(5x) - \ln 3$

(D) $\dfrac{1}{2}\ln(5x) - \ln 3$

6. A curve is defined parametrically by $x = -\ln t$, $y = \cos 2t$ for $t > 0$.
At what approximate value of x does the curve cross the x-axis for the first time as t increases from zero?

(A) -1.7

(B) -1.37

(C) -0.86

(D) 0.24

7. Find the standard deviation of the Bernoulli random variable with the probability distribution represented by the following piecewise function.

$$P(X = x) = \begin{cases} 0.27 & x = 1 \\ 1 - p & x = 0 \end{cases}$$

(A) 0.1971

(B) 0.4440

(C) 0.03884

(D) 0.7768

8. In the cartesian plane, a vector perpendicular to the line $3x + 2y + 1 = 0$ is

(A) $3\underset{\sim}{i} + 2\underset{\sim}{j}$

(B) $-\frac{1}{2}\underset{\sim}{i} + \frac{1}{3}\underset{\sim}{j}$

(C) $2\underset{\sim}{i} - 3\underset{\sim}{j}$

(D) $\frac{1}{2}\underset{\sim}{i} - \frac{1}{3}\underset{\sim}{j}$

9. What is the simplified form of:

$$\frac{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}}{\cot\frac{\theta}{2} + \tan\frac{\theta}{2}} =$$

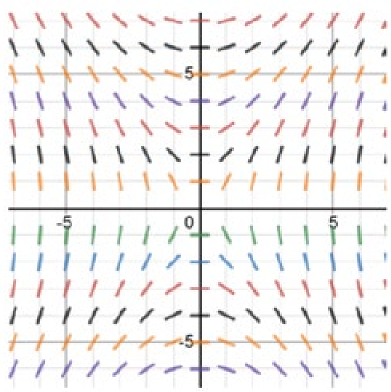
(*A*) $\cos\theta$

(*B*) $\sec\theta$

(*C*) $\tan\theta$

(*D*) $\cot\theta$

10. The slope field represent which of the following equation?



(*A*) $\frac{dy}{dx} = \frac{2x}{y}$

(*B*) $\frac{dy}{dx} = \frac{2y}{x}$

(*C*) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(*D*) $\frac{dy}{dx} = \frac{y^2}{x^2}$

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SECTION II:

Question 11 **Start on a new answer booklet** **15 Marks**

a. A polynomial $P(x)$ when divided by $(x-2)(x+1)$ gives a remainder $(x-b)$. **2**
When this polynomial $P(x)$ is divided by $(x+1)$ it gives a remainder 3.
Find the value of b.

b. A ball is rolling along a horizontal plane has position vector

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} \quad y \geq 0$ and velocity vector $\dot{\underset{\sim}{r}} = \frac{1}{y}\underset{\sim}{i} + (1-y)\underset{\sim}{j}$.

i. The component of velocity in the $\underset{\sim}{j}$ direction gives the differential equation

$\frac{dy}{dt} = 1 - y$. Show that the solution to this differential equation is

$\ln|1-y| = A - t$ Where A is a constant. **2**

ii. Given that the ball is initially at the origin and that the y values are restricted to $0 \leq y < 1$. Find the equation of y in terms of e and t. **1**

iii. Hence or otherwise, find the velocity vector when $t=3$. **2**

c. Using the substitution $u = e^x + 1$ or otherwise, find the exact value **3**

in a single fraction form of $\int_0^1 \frac{e^x}{(1+e^x)^2}\,dx$

d. A missile is shot from the origin O with initial speed of 64 ms^{-1}
at an angle of 30° to the horizontal. The equations of motion are
$\ddot{x} = 0$ and $\ddot{y} = -10$.

i. Show that $x = 32\sqrt{3}\,t$ **1**

ii. Show that $y = 32t - 5t^2$ **2**

iii. What is the cartesian equation of the trajectory of the missile? **2**

Question 12 **Start on a new answer booklet** **15 Marks**

a. Solve $\dfrac{x-5}{2x+1} \geq 1$. **3**

b. Given that $y = e^{2x} + e^{-2x}$

i. Find $\dfrac{d^2y}{dx^2}$ **1**

ii. Determine the values of constants a and b that satisfy the following equation:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 5e^{2x} + e^{-2x}$$ **3**

c. A bag contains 5 identical blue marbles, 6 identical black marbles and 3 identical red marbles. Three marbles are drawn at random. Find the probability that exactly two blue marbles are drawn. (Answer in simplified fraction form) **2**

d. **i.** Find $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ using the substitution $u = 1 - x^2$. **2**

ii. Differentiate $x\cos^{-1} x$ with respect to x. **2**

iii. Hence, find $\displaystyle\int \cos^{-1} x\,dx$. **2**

Question 13 **Start on a new answer booklet** **15 Marks**

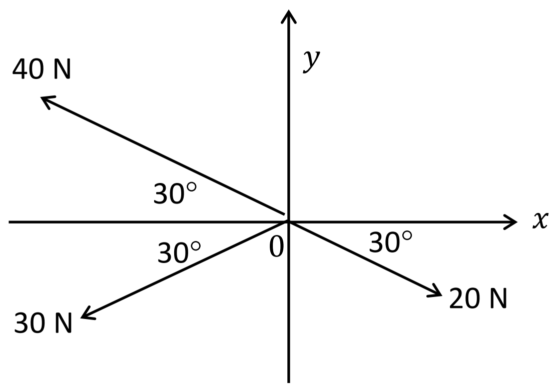
a. Given that $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j}$.

i. Find $\underset{\sim}{u} - \underset{\sim}{v}$. **1**

ii. If $\underset{\sim}{u} - \lambda \underset{\sim}{v} = -2\underset{\sim}{i} - 5\underset{\sim}{j}$. Find λ. **1**

iii. If $a\underset{\sim}{u} + b\underset{\sim}{v} = \underset{\sim}{i}$. Find a and b. **1**

b. The diagram shows three forces acting on an object.



Find the magnitude of the resultant force in Newton and its direction to the nearest degree. **3**

c. Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx$ **3**

d. Find the volume, in terms of π of the solid formed when the area bounded by the curve $y = \frac{1}{2}\log_e x$, the lines $y = -1$ and $y = 2$ and the *y-axis* is rotated about the *y-axis*. **3**

e. Use the compound angles identities to first simplify and then solve $\cos 3x - \cos 2x + \cos x = 0$ for $0 \leq \theta \leq \pi$. **3**

Question 14 **Start on a new answer booklet** **15 Marks**

a. Use mathematical induction to prove that, for any integer $n \geq 1$

$3^{2n+6} - 4^{n+1}$ is divisible by 5. **4**

b. **i.** Prove that $\cot\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1 - \cos\theta}$ **2**

ii. Hence find the exact value with a rationalised denominator of **2**

$\cot\left(\frac{\theta}{2}\right)$ given that $\sin\theta = \frac{5}{6}$ and $\frac{\pi}{2} < \theta < \pi$.

c. A cone is expanding. At a time t seconds, it has a radius r cm.

A perpendicular height $h = r\sqrt{3}$ cm and its surface area is increasing

at a rate of $\left(\frac{\pi}{\sqrt{3}}\right)^{\frac{1}{3}}$ $\text{cm}^2 s^{-1}$.

As the cone expands it remains conical and similar to its original shape.

i. Show that its surface area is $S = 3\pi r^2$ cm^2

and its volume $V = \frac{\sqrt{3}}{3}\pi r^3$ cm^3. **2**

ii. Show that $\frac{dV}{dt} = \frac{\sqrt{3}}{6}V^{\frac{1}{3}}$. **3**

iii. Given that the initial volume of the cone is $27\,000$ cm^3

calculate its volume when $t = 250\sqrt{3}$ seconds. **2**

End of Task

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Solutions.

Student's Name: Multiple Choice. Teacher's Code: Q's 1 – 10

Question Number	Marker use only

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

Saint Ignatius' College
RIVERVIEW

Mathematics

Writing Booklet

Instructions

- Start each question in a new booklet.
- Except for multiple choice answers, you should show relevant mathematical reasoning and/or calculations.
- You may ask for an extra Writing Booklet to answer this question if you need more space.
- If you do not attempt a question you must still hand in a Writing Booklet, with your Name, your Teacher's Name, the Question Number the words 'NOT ATTEMPTED' written clearly on the front cover.
- Write the number of each section part inside the margin at the beginning of each answer.
- Write using black or blue pen.
- Write on the ruled pages only.

You may not remove any Writing Booklets, used or unused, from the examination room.

Start your answer here.

Multiple Choice.

1. $f(x) = \sqrt{x} - 3$

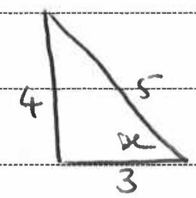
$D: x \geq 0 \quad R: y \geq -3$

$\therefore f^{-1}(x) \quad D: x \geq -3, \quad R: y \geq 0$

(A)

2. Value of $\sin 2x$

$\sin x = 0.8 = \frac{4}{5}$



$\sin 2x = 2 \sin x \cos x$

$= 2 \times \frac{4}{5} \times \left(\frac{-3}{5}\right) = -\frac{24}{5}$

x is obtuse $\therefore \sin 2x = -\frac{24}{5}$

(B)

3. $y = \cos^{-1}(2x)$

$y = \cos^{-1}\left(\frac{2x}{4}\right) + 3$

(C)

4. $y = f(x) + 3$

Moving all pts. up. 3 (+3)

resulting in 3 roots, which is 2 more than the original fn.

(B)

Comments.

$-2 \times f(x)$	1 soln.
$f^{-1}(x)$	1 soln.
$f(x+3)$	1 soln.

5. $f(x) = \frac{3+e^{2x}}{5}$ $\quad f(x) > \frac{5}{3}$

$y = \frac{3+e^{2x}}{5}$

$\therefore x = \frac{3+e^{2y}}{5}$

$5x - 3 = e^{2y}$

$\ln|5x-3| = 2y$

$y = \frac{1}{2}\ln|5x-3|$

(D)

6. x-intercept will occur when $y=0$

$\therefore \cos 2t = 0$

$\therefore 2t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$

$\therefore$ 1st x-intercept at $t = \frac{\pi}{4}$

Sub. $t = \frac{\pi}{4}$, in $x = -\ln t$

$x = -\ln\left(\frac{\pi}{4}\right)$

$\doteq 0.24$ (2dp.)

(D)

7. $\sigma = \sqrt{pq}$

$= \sqrt{0.27 \times 0.73}$

$\doteq 0.4440$

(B)

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8. $3x+2y+1=0 \ (l)$

$2y = -3x-1$

$y = \frac{-3}{2}x - \frac{1}{2}$

$\therefore m_l = \frac{-3}{2}$ $\therefore m_\perp = \frac{2}{3}$

$\therefore 3\underset{\sim}{i} + 2\underset{\sim}{j}$

(A)

9. $\dfrac{\cot\frac{\theta}{2} - \tan\frac{\theta}{2}}{\cot\frac{\theta}{2} + \tan\frac{\theta}{2}}$ Let $\tan\frac{\theta}{2} = t$

$\therefore \cot\frac{\theta}{2} = \frac{1}{t}$

$\dfrac{\frac{1}{t} - t}{\frac{1}{t} + t} = \dfrac{1-t^2}{t} \times \dfrac{t}{1+t^2}$

$= \dfrac{1-t^2}{1+t^2} = \cos\theta$

(A)

10. (A)

You may ask for an extra Writing Booklet if you need more space to answer this question.

Student's Name: Question 11. Teacher's Code: ____________

Question Number	Marker use only
11	

(a)	
(b)	
(c)	
(d)	
(e)	
(f)	
(g)	
(h)	

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Question 11

a. $\cancel{(2x-1)^3}$

a. $P(x) = (x-2)(x+1)Q(x) + (x-b)$

$P(-1) = 3$ ✓ since $P(x) \div (x+1)$ $R(x)=3$

$\therefore P(-1) = (-1-2)(-1+1)Q(x) + (-1-b) = 3$ many didn't recognise this.

$= (-3)(0)Q(x) + (-1-b) = 3$

$-1-b = 3$

$-b = 4$

$\therefore b = -4.$ ✓ evaluating correctly.

b. (i) $\frac{dy}{dt} = 1-y$

$\int dt = \int \frac{dy}{1-y}$ or $t = \int \frac{1}{1-y}\,dy$ ✓ showing this point or equivalent

$t = -\ln|1-y| + A$

$-\ln|1-y| = t - A$ or $\ln|1-y| = A - t$ ✓ integrating

(ii) When $t=0$, $y=0$ $\therefore \ln|1| = A$

i.e $A = 0.$

Since $0 \le y \le 1$ $\ln(1-y) = -t$

$1-y = e^{-t}$ $y = 1-e^{-t}$ ✓ finding the equation.

(iii) When $t=3$, $y = 1 - \frac{1}{e^3}$

Velocity vector when $t=3$

$\dot{r} = \frac{1}{y}\underline{i} + (1-y)\underline{j}$ Given.

$y = 1 - \frac{1}{e^3} = \frac{e^3-1}{e^3}$

$\therefore \frac{1}{y} = \frac{e^3}{e^3-1}$

$1-y = \frac{1}{e^3}$

Sub. the values in $\dot{r}$

$\therefore \dot{r} = \frac{e^3}{e^3-1}\underline{i} + \frac{1}{e^3}\underline{j}$ ✓✓

also allowed $\dot{\underset{\sim}{r}} = \frac{1}{1-e^{-3}}\underset{\sim}{i} + e^{-3}\underset{\sim}{j}$

c. $\int_0^1 \frac{e^x}{(1+e^x)^2}dx$

$u = e^x+1$

$du = e^x dx$

@ $x=0$, $u=2$

@ $x=1$, $u=e+1$ ✓ finding appropriate integral

$\therefore \int_2^{e+1} \frac{du}{u^2}$

$\int_2^{e+1} u^{-2}du = \left[-u^{-1}\right]_2^{e+1}$ ✓ integrating

$= -\left[\frac{1}{u}\right]_2^{e+1} = -\left[\frac{1}{e+1} - \frac{1}{2}\right]$

$= -\left[\frac{2-e-1}{2(e+1)}\right] = -\left[\frac{1-e}{2(e+1)}\right]$

$= \frac{e-1}{2(e+1)}$ (In single fraction form) ✓ finding exact value as a single fraction.

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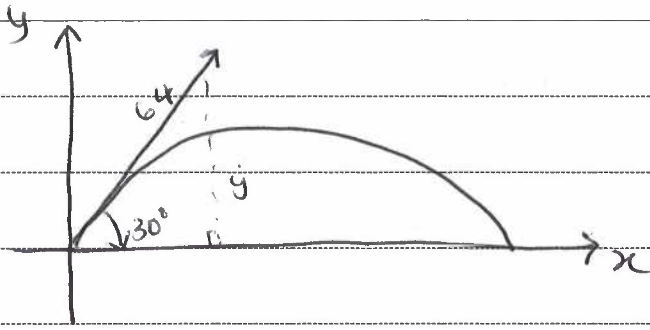
(d)



(i) $a_x = \ddot{x} = 0$

$V_x = \dot{x} = C_1$

At $t=0$, $V_x = 64\cos 30° = 32\sqrt{3}$

$\therefore \dot{x} = 32\sqrt{3}\ ms^{-1}$

$x = 32\sqrt{3}t + C_2$

When $t=0$, $x=0$ $\therefore C_2 = 0$

$\therefore x = 32\sqrt{3}t$.

✓ Showing initial $\dot{x}$ and evaluating constants.

(ii) Vertically $a_y = \ddot{y} = -10$

$V_y = \dot{y} = -10t + C_3$

at $t=0$, $V_y = 64\sin 30° = 32\ ms^{-1} = C_3$

ie $\dot{y} = -10t + 32$

✓ Showing initial $\dot{y}$ and evaluating constant

$\therefore y = -5t^2 + 32t + C_4$

at $t=0$, $y=0$ $\therefore C_4 = 0$

$y = -5t^2 + 32t$

✓ integrating and evaluating constan

(iii) $x = 32\sqrt{3}t$ $\quad t = \dfrac{x}{32\sqrt{3}}$ ✓ making t subject

$y = -5t^2 + 32t$

$= -5\left[\dfrac{x}{32\sqrt{3}}\right]^2 + 32\left[\dfrac{x}{32\sqrt{3}}\right]$

$y = -\dfrac{5x^2}{3072} + \dfrac{\sqrt{3}x}{3}$

✓ Finding cartesian equation

(many students did not know what this means)

You may ask for an extra Writing Booklet if you need more space to answer this question.

QUESTION 12

a) $\dfrac{x-5}{2x+1} \geq 1 \qquad x \neq -\frac{1}{2}$

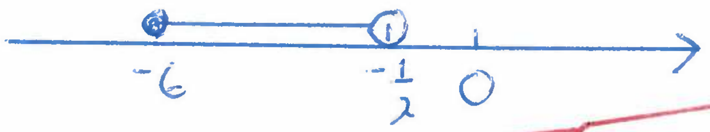
$\dfrac{x-5}{2x+1} \times (2x+1)^2 \geq 1 \times (2x+1)^2$ ← 1mk

Too many did not spot that $x \neq -\frac{1}{2}$.

$(x-5)(2x+1) \geq (2x+1)^2$

$(2x+1)^2 - (x-5)(2x+1) \leq 0$

$(2x+1)(x+6) \leq 0$



$-6 \leq x < -\frac{1}{2}$

1mk for each inequality.

b) i) $y = e^{2x} + e^{-2x}$

Well done

$y' = 2e^{2x} - 2e^{-2x}$

$y'' = 4e^{2x} + 4e^{-2x}$ ← 1mk

ii) $y'' + ay' + by = 5e^{2x} + e^{-2x}$

$$\begin{aligned}
RHS &= 4e^{2x} + 4e^{-2x} + a(2e^{2x} - 2e^{-2x}) + b(e^{2x} + e^{-2x}) \\
&= 4e^{2x} + 4e^{-2x} + 2ae^{2x} - 2ae^{-2x} + be^{2x} + be^{-2x} \\
&= e^{2x}(4 + 2a + b) + e^{-2x}(4 - 2a + b)
\end{aligned}$$

1mk for correct setting out

$\therefore \quad 5 = 4 + 2a + b \qquad\qquad 1 = 4 - 2a + b$

$2a + b = 1 \quad —① \qquad 2a - b = 3 \quad —②$

$① + ② \quad 4a = 4 \qquad 2 - b = 3$

$a = 1 \qquad b = -1$

1mk each for
$a = 1$
$b = -1$

---

c) 5 Blue 6 Black 3 Red.

Exactly 2 Blue in 3 $= {}^5C_2 \times {}^9C_1$

$= 90$

arrangments of 3 from 14 $= {}^{14}C_3$

$= 364$

$\therefore P(e) = \dfrac{90}{364}$ OR $\dfrac{45}{182}$

OR CORRECT FROM TREE DIAGRAM

Poorly don

1mk each

OR

---

d) i) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$

$u = 1 - x^2 \qquad\qquad \displaystyle = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$\dfrac{du}{dx} = -2x \qquad\qquad \displaystyle = -\frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$

$du = -2x\,dx \qquad\qquad = -\sqrt{u} + c$

$= -\sqrt{1-x^2} + c$

1mk for set up

1mk

ii) $\frac{d}{dx} x \cos^{-1} x = x \cdot \frac{-1}{\sqrt{1-x^2}} + \cos^{-1} x$

$= \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$

Ink for each "part"

iii) $\int \cos^{-1} x \, dx = ???$

From (ii)

$\frac{d}{dx} x \cos^{-1} x = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$

$\cos^{-1} x = \frac{d}{dx} x \cos^{-1} x + \frac{x}{\sqrt{1-x^2}}$

$\therefore \int \cos^{-1} x = x \cos^{-1} x + \int \frac{x}{\sqrt{1-x^2}} dx$

$= x \cos^{-1} x - \sqrt{1-x^2} + c$

Well done by most

Ink for each "part"

Start your answer here.

Question 13

a. $\underline{u} = 2\underline{i}+3\underline{j}$ $\quad \underline{v} = \underline{i}+2\underline{j}$ * Well answered

(i) $\underline{u}-\underline{v} = 2\underline{i}-\underline{i}+3\underline{j}-2\underline{j}$

$= \underline{i}+\underline{j}$

(ii) $\underline{u}-\lambda\underline{v} = -2\underline{i}-5\underline{j}$

$\therefore 2\underline{i}+3\underline{j}-\lambda\underline{i}-\lambda 2\underline{j} = -2\underline{i}-5\underline{j}$

$(2-\lambda)\underline{i}+(3-2\lambda)\underline{j} = -2\underline{i}-5\underline{j}$

$\therefore 2-\lambda = -2$ $\quad$ Check:

$\lambda = 4$ $\quad$ $3-2\lambda = -5$

$-2\lambda = -8$

$\lambda = 4.$ * Well answered

$\therefore \lambda = 4.$

(iii) $a\underline{u}+b\underline{v} = \underline{i}$

$a(2\underline{i}+3\underline{j})+b(\underline{i}+2\underline{j}) = \underline{i}+0\underline{j}$

$2a\underline{i}+b\underline{i}+3a\underline{j}+2b\underline{j} = \underline{i}+0\underline{j}$

$2a+b=1$ $\quad$ $3a+2b=0$

$b=1-2a$ $\quad$ $3a+2(1-2a)=0$

$3a+2-4a=0$

$-a=-2$

$\therefore b = 1-2(2)$ $\quad$ $a=2$

$b=-3$

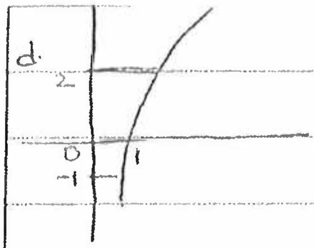
$\therefore a=2, \ b=-3$

* Well answered.

b. Method: write each vector in the form of $a\underline{i} + b\underline{j}$ and then add for Resultant Force	* Some students adopted a visual, geometric approach and made mistakes.
40N : $40\cos 150^\circ \underline{i} + 40\sin 150^\circ \underline{j} = -20\sqrt{3}\underline{i} + 20\underline{j}$	
30N : $30\cos 210^\circ \underline{i} + 30\sin 210^\circ \underline{j} = -15\sqrt{3}\underline{i} - 15\underline{j}$	
20N : $20\cos(-30)\underline{i} + 20\sin(-30)\underline{j} = 10\sqrt{3}\underline{i} - 10\underline{j}$	* mistakes were made when calculating various sin and cos values.
$R = (-20\sqrt{3} - 15\sqrt{3} + 10\sqrt{3})\underline{i} + (20 - 15 - 10)\underline{j}$	
$= -25\sqrt{3}\underline{i} - 5\underline{j}$	
Magnitude: $R: \sqrt{(-25\sqrt{3})^2 + (-5)^2} = \sqrt{1900}$	
$= 10\sqrt{19}$ N	
Direction: $\tan\theta = \frac{-5}{-25\sqrt{3}} = \frac{1}{5\sqrt{3}}$	* Follow on marks were awarded if you used correct method to calculate length and value of angle.
$\therefore \theta = \tan^{-1}\left(\frac{1}{5\sqrt{3}}\right) = 186^\circ 35'$	
$\doteq 187^\circ$.	
Hence, the resultant vector has magnitude $10\sqrt{19}$ N and forms an angle of 187° with the positive direction of the x-axis.	
c. $\int_{\pi/4}^{\pi/2} \sin^2 x\,dx = \frac{1}{2}\int_{\pi/4}^{\pi/2} 1 - \cos 2x\,dx$	* Most students adopted correct approach but some made silly errors with signs and when subtracting
$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_{\pi/4}^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - \left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2}\right)\right]$	
$= \frac{1}{2}\left[\frac{\pi}{2} - 0 - \frac{\pi}{4} + \frac{1}{2}\right] = \frac{1}{2}\left[\frac{\pi}{4} + \frac{1}{2}\right]$	
OR $= \frac{\pi}{8} + \frac{1}{4}$	

Additional writing space on back page.

d.



$$V = \pi \int_a^b x^2 \, dy$$

$$= \pi \int_{-1}^{2} (e^{2y})^2 \, dy$$

$$= \pi \int_{-1}^{2} e^{4y} \, dy \qquad = \frac{\pi}{4}\Big[e^{4y}\Big]_{-1}^{2}$$

$$= \frac{\pi}{4}\left(e^{8} - e^{-4}\right) u^3$$

$y = \frac{1}{2} \log_e x$

$2y = \log_e x$

$e^{2y} = x$

* Well answered

* Some students tried to calculate the answer using $V = \pi \int y^2 \, dx$ which is incorrect.

(e) $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\therefore \cos 3x + \cos x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$

$= 2\cos 2x \cos x$

$\therefore \cos 3x + \cos x - \cos 2x = 0$

is $2\cos 2x \cos x - \cos 2x = 0$

$\cos 2x (2\cos x - 1) = 0$

$\cos 2x = 0 \qquad \cos x = \frac{1}{2}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{\pi}{3}$ 1st 2 Quad only.

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$

Soln: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{3}$

* Reasonably well answered

* Some students included solutions outside range

* If you used identities which were relevant to try and prove the statement but did not finish proof you received 1 mark.

You may ask for an extra Writing Booklet if you need more space to answer this question.

14a) R.T.P. $3^{2n+6} - 4^{n+1} / 5 \quad n \geq 1$

<u>let $n=1$</u>

$LHS = 3^{2(1)+6} - 4^{1+1}$

$= 3^8 - 4^2$

$= 6545$

$= 5 \times 1309$ ✓

$\therefore$ true for $n=1$

<u>Assume true for $n=k$</u>

ie $3^{2k+6} - 4^{k+1} = 5M$, M an integer

or

$3^{2k+6} = 5M + 4^{k+1}$

<u>let $n=k+1$</u>

R.T.P $3^{2(k+1)+6} - 4^{(k+1)+1} = 3^{2k+8} - 4^{k+2} = 5N$, N an integer.

$LHS = 3^{2k+8} - 4^{k+2}$

$= 3^{2k+6} \cdot 3^2 - 4 \cdot 4^{k+1}$

$= 9 \times 3^{2k+6} - 4 \cdot 4^{k+1}$ ✓

$= 9(5M + 4^{k+1}) - 4 \cdot 4^{k+1}$ from assumption ✓

$= 45M + 9 \times 4^{k+1} - 4 \times 4^{k+1}$

$= 45M + 5 \times 4^{k+1}$

$= 5(9M + 4^{k+1})$

$= 5N$, N an integer ✓

$\therefore$ True by principle of Mathematical Induction b.) i.) Prove $\cot\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1-\cos\theta}$

$$RHS = \frac{\sin\left(2\left(\frac{\theta}{2}\right)\right)}{1-\cos\left(2\frac{\theta}{2}\right)}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-\left(1-2\sin^2\frac{\theta}{2}\right)} \checkmark$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} \checkmark$$

$$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \cot\frac{\theta}{2}$$

OR

let $\tan\frac{\theta}{2} = t$ $\therefore \cot\left(\frac{\theta}{2}\right) = \frac{1}{t}$

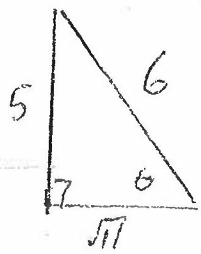
$\sin\theta = \frac{2t}{1+t^2}$ $\cos\theta = \frac{1-t^2}{1+t^2}$

$$RHS = \frac{\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}} \checkmark = \frac{2t}{1+t^2-1+t^2} \checkmark$$

$$= \frac{2t}{2t^2}$$

$$= \frac{1}{t}$$

(ii) $\sin\theta = \frac{5}{6}$ $\frac{\pi}{2} \le \theta \le \pi$



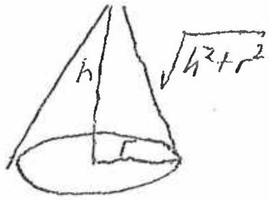
$$\therefore \cot\left(\frac{\theta}{2}\right) = \frac{\frac{5}{6}}{1-\left(-\frac{\sqrt{11}}{6}\right)} \checkmark$$

$$= \frac{5}{6+\sqrt{11}} \times \frac{6-\sqrt{11}}{6-\sqrt{11}}$$

$$= \frac{30-5\sqrt{11}}{36-11}$$

$$= \frac{30-5\sqrt{11}}{25} = \frac{6-\sqrt{11}}{5} \checkmark$$

c.)



$h = r\sqrt{3}$

$$\frac{dA}{dt} = \left(\frac{\pi}{\sqrt{3}}\right)^{1/3}$$

(i) $S = \pi r l + \pi r^2$

$l = \sqrt{h^2+r^2} = \sqrt{3r^2+r^2} = 2r$ ✓

$\therefore S = \pi r(2r) + \pi r^2$

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 r\sqrt{3} = \frac{\sqrt{3}}{3}\pi r^3$ ✓

(ii) $\frac{dV}{dt} = \frac{dV}{dA} \times \frac{dA}{dt}$

$= \frac{dV}{dr} \times \frac{dr}{dA} \times \frac{dA}{dt}$

$= \sqrt{3}\pi r^2 \times \frac{1}{6\pi r} \times \left(\frac{\pi}{\sqrt{3}}\right)^{\frac{1}{3}}$

$= \frac{\sqrt{3}}{6} \times r \times \left(\frac{\pi}{\sqrt{3}}\right)^{\frac{1}{3}}$ ✓

$= \frac{\sqrt{3}}{6} \times \left(\frac{\sqrt{3}}{3}\pi r^3\right)^{\frac{1}{3}}$ ✓ $= \frac{\sqrt{3}}{6} V^{1/3}$

(iii) $t = 0 \quad V = 27{,}000$

$\frac{dV}{dt} = \frac{\sqrt{3}}{6} V^{1/3}$

$V^{-\frac{1}{3}} dV = \frac{\sqrt{3}}{6} dt$

$\frac{3V^{2/3}}{2} = \frac{\sqrt{3}}{6} t + c$

$\therefore c = \frac{3(27{,}000)^{2/3}}{2} = 1350$

$V = \left(\frac{\sqrt{3}}{9} t + 900\right)^{3/2}$ ✓

$t = 250\sqrt{3} \quad V = 30835.51$ ✓